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NUMERICAL SIMULATION OF RECEPTIVITY OF A HYPERSONIC BOUNDARY LAYER TO ACOUSTIC DISTURBANCES

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Direct numerical simulations of the evolution of disturbances in a viscous shock layer on a flat plate are performed for a free-stream Mach number $M_\infty = 21$ *and Reynolds number* $Re_L = 1.44 \cdot 10^5$*. Unsteady Navier–Stokes equations are solved by a high-order shock-capturing scheme. Processes of receptivity and instability development in a shock layer excited by external acoustic waves are considered. Direct numerical simulations are demonstrated to agree well with results obtained by the locally parallel linear stability theory (with allowance for the shock-wave effect) and with experimental measurements in a hypersonic wind tunnel. Mechanisms of conversion of external disturbances to instability waves in a hypersonic shock layer are discussed.*

Key words: *direct numerical simulation, Navier–Stokes equations, hypersonic shock layer, acoustic disturbances.*

Introduction. Understanding the mechanisms of receptivity and instability of a viscous shock layer is necessary for developing effective methods of controlling the laminar–turbulent transition in hypersonic flight. When a flying vehicle moves with a high velocity in the upper layers of the atmosphere, the viscous shock layer regime is extended to a significant distance from the leading edges. Origination and evolution of disturbances in the shock layer may be significantly different from those in supersonic near-wall flows with moderate Mach numbers ($M_\infty < 10$) [1–4] and have been little studied yet. A theoretical study of such flows is difficult because of the interaction of disturbances with the shock wave (SW), significant nonparallelism of the flow, and velocity slip and temperature jump on the wall. Possibilities of wind-tunnel modeling of receptivity and disturbance evolution in a hypersonic shock layer are limited; in particular, wind-tunnel experiments cannot ensure real-flight Reynolds numbers and flow enthalpy. Numerical simulations can fill this gap. There are some recent publications [5–8], where the problems of receptivity and evolution of disturbances in supersonic and moderate hypersonic flows have been solved by means of direct numerical simulations (DNS) on the basis of full unsteady Navier–Stokes equations. This approach allows obtaining detailed information on the disturbance field, which is necessary for verification of theoretical models and comparisons with measurement data. The studies performed up to now, however, involved flow parameters more typical of the boundary layer (where the SW is rather far from the upper edge of the viscous flow) rather than of the shock layer.

Results of a parametric research of shock-layer interaction with external acoustic waves propagating at different angles to the flow with an extremely high Mach number ($M_\infty = 21$) and a moderate Reynolds number ($Re_L = 1.44 \cdot 10^5$) are described in the present paper. Interaction of acoustic perturbations of the external flow (slow and fast modes) with the shock layer is simulated by solving two-dimensional Navier–Stokes equations. The

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computed results are compared with characteristics of density fluctuations measured at identical flow parameters in a T-327A hypersonic nitrogen-driven wind tunnel of the Khristianovich Institute of Theoretical and Applied Mechanics of the Siberian Division of the Russian Academy of Sciences.

Formulation of the Problem. We consider a hypersonic flow past an infinitely thin flat plate of length $L = 24$ cm mounted at zero incidence to the incoming flow. Two-dimensional Navier–Stokes equations written in the form of conservation laws are solved with a high-accuracy shock-capturing scheme. (The numerical method is described in detail in [9, 10].) The gas is assumed to be perfect and have constant specific heats; the gas viscosity is calculated by Sutherland's formula with parameters corresponding to nitrogen. The boundary conditions of velocity slip and temperature jump are set on the plate surface. The temperature of the plate surface is assumed to be constant: $T_w = 300$ K. As was shown in [9], the mean flow computed in such a formulation is in good agreement with the measured total pressure, density, and Mach number. The rarefaction effects in the problem considered are fairly significant: for $x/L = 0.1$, the slip velocity on the plate surface is approximately 17% of the free-stream velocity, and the corresponding value at the trailing edge of the plate is 7%.

The viscous shock layer is excited by external acoustic waves. In the numerical solution, disturbances in the form of a plane monochromatic wave were imposed onto the uniform flow at the input boundary of the computational domain:

$$\begin{pmatrix} u' \\ v' \\ p' \\ \rho' \end{pmatrix} = A \begin{pmatrix} \pm\cos\theta \\ \mp\sin\theta \\ 1 \\ 1 \end{pmatrix} \exp\,[i(k_x x + k_y y - \omega t)].$$

Here u', v', p', and ρ' are the fluctuations of the streamwise and transverse velocity, pressure, and density, respectively, θ is the angle of propagation of the external acoustic wave, A is the disturbance amplitude, t is the time, and $k_x = k\cos\theta$ and $k_y = -k\sin\theta$ are the components of the wave vector related to the frequency $\omega = 2\pi f L/c_\infty$ by the dispersion expression $k = \omega/(\mathrm{M}_\infty\cos\theta \pm 1)$; the upper (lower) sign refers to a fast (slow) acoustic wave. In recording the relations in dimensionless form, the density and temperature disturbances are normalized to their free-stream values, velocity disturbances to the velocity of sound c_∞, and pressure disturbances to $\rho_\infty c_\infty^2$. These disturbances simulate natural free-stream perturbations, which are always present in the wind tunnel. The amplitude, frequency, and phase characteristics of density fluctuations of natural disturbances were measured in experiments [9] with the use of electron-beam fluorescence of nitrogen by the method described in [11]. For comparisons with the measured data, the results of numerical simulations in [9] were averaged over the frequency band of external disturbances corresponding to the test conditions; the data of numerical simulations in the present work were not averaged.

In the computations, the disturbance amplitude A was assumed to be 0.028. The problem was found to be linear in the range of external disturbance amplitudes at least up to $A = 0.04$. The angles of incidence of acoustic waves onto the plate θ were chosen in the range from $-10°$ to $45°$. The angles smaller than $-10°$ were not considered, because the shock layer in this case is in an "acoustic shadow" of the plate, and the problem of diffraction of acoustic waves in the plate "tip" was not analyzed.

After introduction of disturbances, the Navier–Stokes equations were integrated until the unsteady solution reached a steady periodic regime. The computational grid was uniform and had $N_x = 1050$ cells in the streamwise direction and $N_y = 240$ in the crossflow direction. Up to 20 processors of the Siberian Supercomputer Center were used in the computations.

Computation Results. The results described in the present paper were obtained for mean flow parameters corresponding to test conditions in the hypersonic wind tunnel: $\mathrm{M}_\infty = 21$, $\mathrm{Re}_{L\infty} = 1.44 \cdot 10^5$, and $T_w/T_0 = 0.25$.

Results of direct numerical simulations of the evolution of disturbances in the shock layer on the plate for $f = 38.4$ kHz and $\theta = 0$ can be compared with computations by the locally parallel linear stability theory with allowance for the shock-wave effect [3, 4] and with the data of [9]. The distributions of the amplitude spectra of density fluctuations along the coordinate normal to the plate surface in cross sections along the model axis were obtained in experiments. Based on these data, the growth rates of disturbances in the shock layer were calculated.

In the present work, we compare the growth rates α_i and phase velocities C_x of disturbances on the SW with computations within the framework of the linear stability theory with allowance for the SW effect, which were performed in [4] and showed that the SW presence near a viscous boundary layer shifts the maximum of density fluctuations on the SW (Fig. 1). It is seen in Fig. 1 that the DNS results for the slow mode are in good agreement

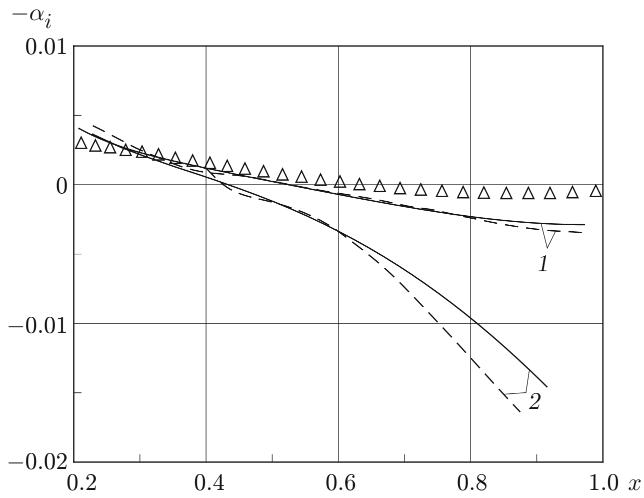

Fig. 1. Growth rate of density fluctuations on the SW versus the streamwise coordinate: the dashed and solid curves are the results obtained by the locally parallel stability theory with allowance for the SW effect [3, 4] and direct numerical simulations, respectively; curves 1 and 2 refer to the slow and fast modes, respectively; the points are the experimental data [9].

with experimental measurements and data of the linear theory; the difference is observed only in an immediate vicinity of the trailing edge of the plate; for the fast mode, the disagreement begins closer to the leading edge.

The phase velocities of density fluctuations are compared in Fig. 2. The fluctuations are seen to predominantly propagate with the free-stream velocity. This means that vortex disturbances arise in the shock layer. Near the leading edge of the flat plate, the phase velocities obtained by the linear stability theory [4] are significantly different from the DNS data. A possible reason is that the linear stability theory [3, 4] ignores nonparallelism of the mean flow, which is fairly noticeable in the vicinity of the leading edge.

Figure 3 shows the computed field of the mean density (Fig. 3a) and instantaneous fields of density fluctuations for the slow (Figs. 3b, 3d, 3f, and 3h) and fast (Figs. 3c, 3e, 3g, and 3i) modes of external acoustic disturbances with a frequency $f = 38.4$ kHz and with different angles of incidence θ. Under the action of external waves, two regions of the most intense density fluctuations are formed in the shock layer: they are aligned along the SW and the line of the greatest transverse gradient of the mean density (cf. Fig. 3a). With increasing angle θ, the maximum of the density fluctuations at the boundary-layer edge decreases both for slow and fast modes of external disturbances. It can also be noted that structures similar to oblique waves start forming between the SW and the boundary-layer edge as the angle θ increases (see Figs. 3h and 3i). This fact can be explained within the framework of the linear theory of SW-disturbance interaction [12]. According to this theory, an acoustic disturbance behind the SW can arise only in the range of angles of propagation of external disturbances limited by critical values. Figure 4 shows the regions of existence of different modes in the shock layer during interaction of slow (Fig. 4a) and fast (Fig. 4b) acoustic waves with the shock wave on the flat plate at $M_\infty = 21$, which were constructed by the theory [12]. The abscissa axis shows the angles φ between the SW and the flow, and the ordinate axis shows the angles θ of propagation of disturbances in the free stream. Both entropy-vortex and acoustic disturbances can arise in regions I and III behind the SW, while it is only entropy-vortex disturbances that are generated in region II. It follows from Fig. 4 that disturbances of the entropy-vortex mode only arise behind the SW for angles of propagation of the slow acoustic wave from $-10°$ to $30°$ and angles of the fast acoustic wave from 0 to $45°$; acoustic disturbances cannot penetrate through the SW (they decay exponentially behind the SW front). For angles of propagation of the slow acoustic wave $\theta = 45°$ and fast acoustic wave $\theta > 0$, both vortex and acoustic disturbances can pass behind the SW; therefore, structures similar to oblique waves are formed. Acoustic waves generate significantly less intense density fluctuations in the shock layer than vortex disturbances do (the latter lead to oscillations of the mean density profile as a whole); thus, the intensity of density fluctuations decreases if the acoustic mode dominates in the shock layer.

It was shown [10] that an increase in frequency of external acoustic disturbances initiates periodic variations of the amplitude of density fluctuations on the SW in the streamwise direction. The characteristic period of variations

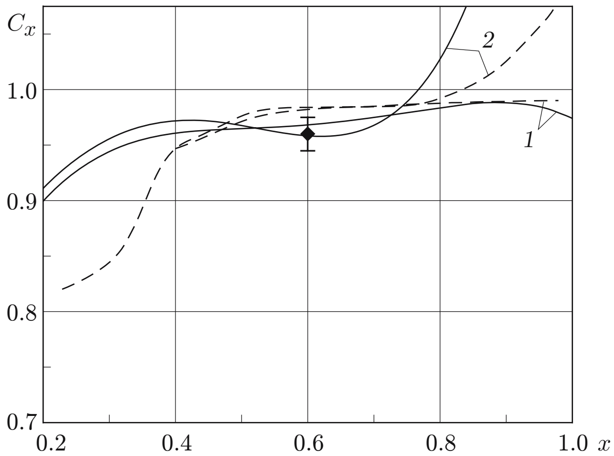

Fig. 2. Phase velocity of density fluctuations on the SW versus the streamwise coordinate: the point is the experimentally measured phase velocity of fluctuations in the shock layer on the flat plate (the remaining notation the same as in Fig. 1).

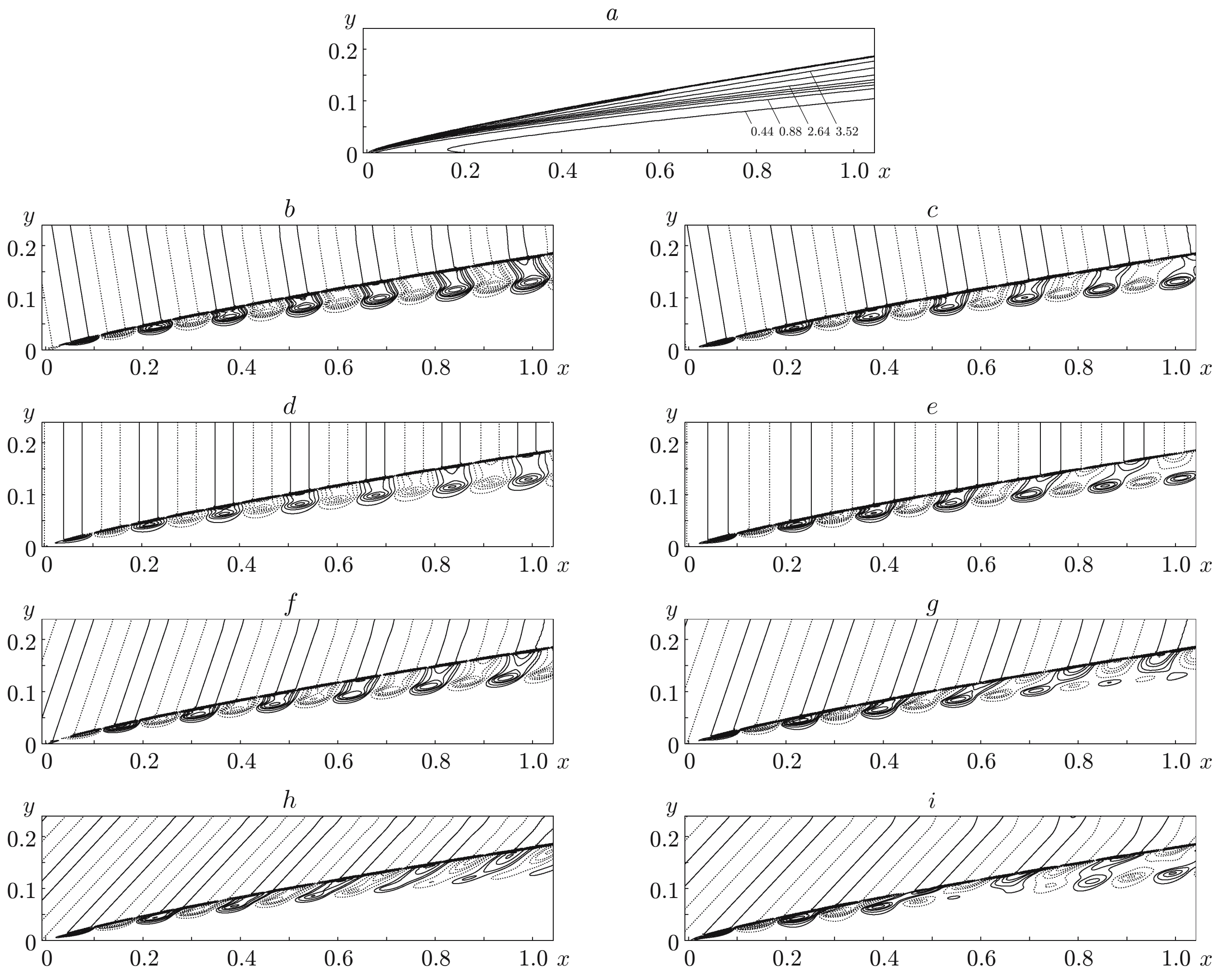

Fig. 3. Mean density field (a) and fields of density fluctuations induced by the slow (b, d, f, and h) and fast (c, e, g, and i) acoustic waves for $f = 38.4$ kHz and $\theta = -10$ (b and c), 0 (d and e), 20 (f and g), and 45° (h and i).

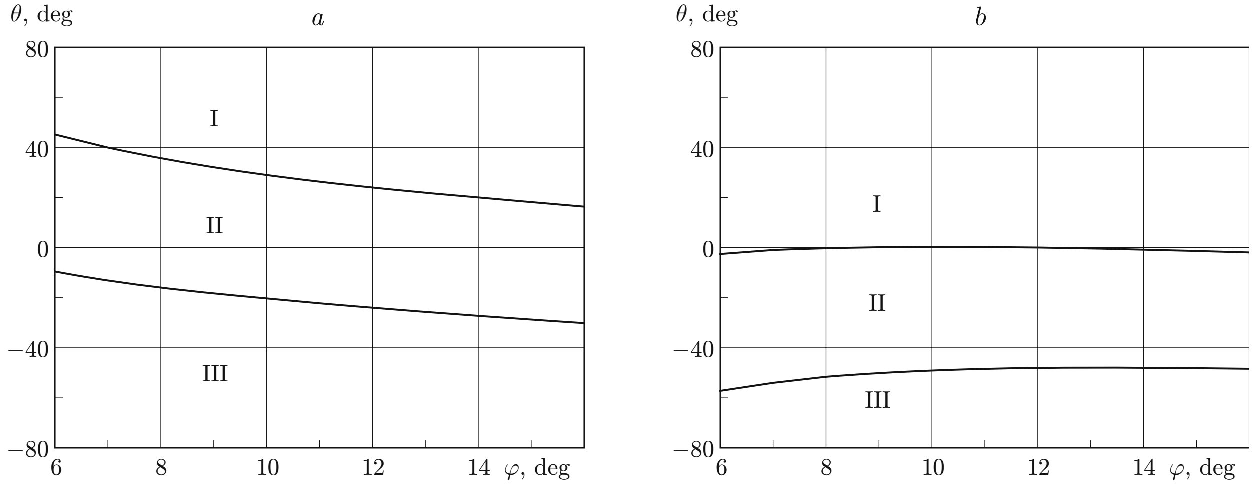

Fig. 4. Regions of existence of different modes in the shock layer during interaction of slow (a) and fast (b) acoustic waves with the SW on the flat plate ($M_\infty = 21$ and $\theta = 0$), as predicted by the theory [12]: vortex and fast acoustic modes (I), vortex mode (II), and vortex and slow acoustic modes (III).

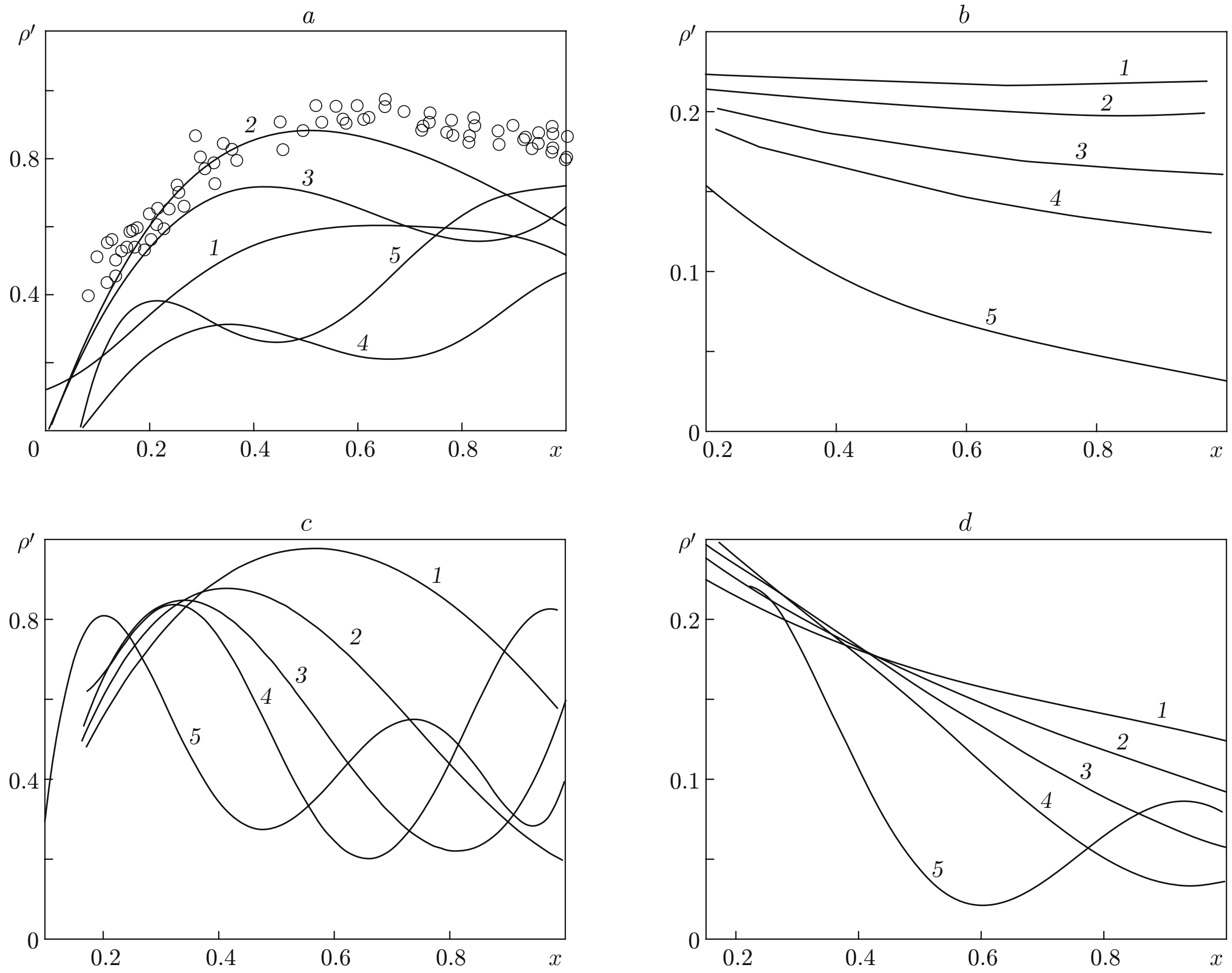

Fig. 5. Amplitude of density fluctuations versus the streamwise coordinate on the SW (a and c) and on the boundary-layer edge (b and d) for $f = 38.4$ kHz and different angles of propagation: the data are given for a slow acoustic wave (a and b) and for a fast acoustic wave (c and d); the points are the experimental data of [9]; $\theta = -10$ (1), 0 (2), 10 (3), 20 (4), and 45° (5).

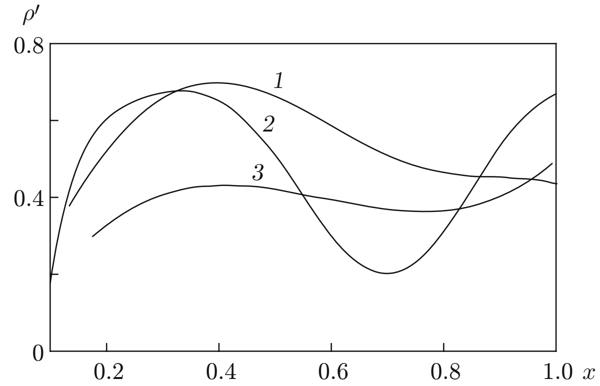

Fig. 6. Amplitude of density fluctuations on the SW versus the streamwise coordinate for $f = 50$ kHz: slow external acoustic wave (1), fast external acoustic wave (2), and external vortex mode (3).

and the maximum amplitude of density fluctuations on the SW are inversely proportional to the frequency of external disturbances and depend only weakly on the mode of external disturbances (slow or fast). Such a dependence of the maximum amplitude of fluctuations on the SW on the frequency of external disturbances is caused by nonlinearity of the problem in the theory of SW-disturbance interaction [12]. Origination of streamwise variations of the amplitude of density fluctuations on the SW, however, demands explanation, in particular, because a similar effect is also observed with increasing angle of propagation of external acoustic disturbances.

Figure 5 shows the variations of the amplitude of density fluctuations along the plate on the SW and boundary-layer edge for the slow and fast modes of external disturbances for $f = 38.4$ kHz. As the angle θ increases, the streamwise scale of variations decreases for both modes, without any significant changes in the maximum amplitude of fluctuations. In the case of the slow mode of external disturbances, the amplitude of density fluctuations at the boundary-layer edge remains essentially unchanged along the plate and only decreases in the absolute value with increasing angle θ (Fig. 5b). For the fast mode of external disturbances, a similar trend is observed for fluctuations on the SW (Fig. 5c). Streamwise variations of the amplitude of density fluctuations also arise on the boundary-layer edge for $\theta > 20°$ (Fig. 5d). The decrease in the amplitude of density fluctuations on the boundary-layer edge (see Figs. 5b and 5d) can also be explained within the theory [12]. According to the data in Fig. 4, as the angle θ increases, the disturbances in the shock layer smoothly transform from the purely entropy-vortex mode to a combination of the entropy-vortex and fast acoustic modes. The fraction of acoustic disturbances continuously increases, which reduces the amplitude of density fluctuations, because density fluctuations generated by acoustic waves in the shock layer are significantly less intense than those generated by vortex disturbances.

Possibly, the streamwise variations of the amplitude of density fluctuations on the SW along the plate are caused by interference of acoustic and vortex disturbances arising behind the SW. Interference of secondary acoustic waves generated in the shock layer by vortex disturbances is also possible for small angles θ.

To check the assumption of wave interference in the shock layer, computations were performed for external vortex disturbances with an elevated frequency (Fig. 6). External vortex disturbances were found to generate approximately the same variations of the amplitude of density fluctuations on the SW as the external acoustic disturbances of the slow and fast modes. This fact confirms the assumption that the streamwise variations of the amplitude on the SW are caused by the spatial structure of disturbances emerging in the shock layer rather than by the mode of external disturbances. This phenomenon, however, requires further investigations.

Conclusions. The process of excitation of instability of a hypersonic viscous shock layer by external acoustic waves is studied by means of direct numerical simulations. The modeling results for disturbances propagating in the streamwise direction are verified by comparisons with the data of the locally parallel linear stability theory and experimental results.

A parametric study of receptivity is performed for disturbances incident onto a viscous shock layer at different angles.

A typical feature of the evolution of disturbances in a viscous shock layer is a periodic change in their amplitude along the streamwise coordinate with increasing angle of propagation of external acoustic waves. It is shown that these variations are not related to the modes of external disturbances.

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